NONISOTHERMAL FILM FLOW OF A MAGNETIZABLE LIQUID

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The problem of the flow of a nonisothermal magnetizable liquid with a free surface in a nonuniform magnetic field is formulated and investigated theoretically by considering a specific example.

Film flows of a magnetizable liquid in a magnetic field have a number of unique features. The magnetic field makes it possible to increase the stability of film flow [1], to ensure forced wave formation on the free surface should the need arise [2], and to control the flow rate and the film thickness within certain limits [3]. In addition, when the magnetic forces are much larger than the gravitational it is possible in practice to ignore the latter completely, thus ensuring the operation of a film device in any orientation in space, and also under zero gravity. The possibility of producing magnetic fields of various spatial configurations leads to a variety of stable forms of films of magnetizable liquid and their flows. Thus, e.g., in the absence of gravitational forces the free surface of a magnetizable liquid in the field of a cylindrical conductor carrying a current has the form of a cylinder coaxial with the conductor; this was investigated for stability in [4].

Isothermal liquids were studied in the papers mentioned above. We take account of the fact that the magnetization and surface tension of the liquid are not constant, and also consider a class of flows in a direction perpendicular to the action of the magnetic force, i.e., in a direction perpendicular to the pressure gradient in the liquid. Thus, in the present case the mechanisms leading to the motion of the liquid are thermomagnetic, thermocapillary, and the fall of the level of the liquid surface relative to the equilibrium position. We note that variations of the magnetization of the liquid and the surface tension can arise not only because of the liquid is not isothermal, but also as a result of concentration gradients of various admixtures.

We consider steady film flow of a weightless, incompressible, nonconducting, magnetizable liquid along a cylindrical conductor of radius R carrying a current I which produces an axisymmetric magnetic field $H_\varphi(r)$. The problem in this case has cylindrical symmetry, and we seek its solution in cylindrical coordinates (r, φ, z) with the z axis along the axis of the conductor. The free surface of the film flowing along the conductor also has axial symmetry, and its coordinates are denoted by ξ(z). It is assumed that the curvature of the free surface in a longitudinal section is small so that it is possible to neglect the flow of the liquid in the radial direction and the z dependence of the only nonzero axial component of the velocity v. This same condition permits the assumption that the magnetization M and the surface tension α are given functions solely of the z coordinate along which, e.g., a temperature gradient may be maintained. The latter is equivalent to the approximation of [5] in the problem of plane-parallel flow of an ordinary liquid under the action of thermocapillary and thermogravitational forces. Maxwell's equations in such an arrangement have an exact solution over all of space $\vec{H} = [0, H_\varphi = H = I/2\pi r, 0]$, and within the liquid, in addition $\vec{M} = [0, M_\varphi = M(z), 0]$. The magnetic field intensity and the magnetization vectors have only azimuthal components tangent to the free surface of the liquid at any point. Henceforth the dependence of M on H is eliminated by assuming that the liquid is magnetized to saturation.

The problem posed is described by the equations of ferrohydrodynamics, which in the present case have the form

$$\frac{\partial P}{\partial z} = \eta \frac{1}{r} \frac{d}{dr} \left(r \frac{dv}{dr} \right), \tag{1}$$

$$\frac{\partial P}{\partial r} = \mu_0 M \frac{dH}{dr}. \tag{2}$$

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Here the pressure jump at the surface of the liquid is determined only by the capillary forces $p - p_0 = \alpha(R_1^{-1} + R_2^{-1})$ at $r = \xi$. The pressure p_0 of the gas above the surface is assumed constant, and the principal radii of curvature R_1 and R_2 are respectively, $R_1 = \xi$ and $R_2 = \infty$. Taking account of this in the above formulation, the solution of Eq. (2) gives the pressure distribution in the liquid

$$p = p_0 + \alpha/\xi + \mu_0 M[H(r) - H(\xi)] = P_0 + \frac{\alpha}{\xi} + \frac{\mu_0 MI}{2\pi}\left(\frac{1}{r} - \frac{1}{\xi}\right), \qquad (3)$$

and by using this we rewrite Eq. (1) for the flow velocity of the liquid in the film

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dv}{dr}\right) = \frac{\mu_0 I}{2\pi\eta}\left(\frac{dM}{dz}\right)\frac{1}{r} - \frac{1}{\eta}\frac{d}{dz}\left[\left(\frac{\mu_0 IM}{2\pi} - \alpha\right)\frac{1}{\xi}\right]. \qquad (4)$$

The boundary conditions for the velocity consist in the no-slip condition on the solid boundary and the balance of tangential stresses on the free boundary

$$v = 0 \quad \text{at} \quad r = R, \quad \eta\frac{dv}{dr} = \frac{d\alpha}{dz} \quad \text{at} \quad r = \xi. \qquad (5)$$

In addition, the condition for a constant flow rate of the liquid

$$2\pi\int_R^\xi rv\,dr = Q \qquad (6)$$

determines the shape of the free surface.

The solution of problem (4), (5) has the following form:

$$v = \frac{\mu_0 I}{2\pi\eta}\left[\frac{dM}{dz}\left(r - R - \xi\ln\frac{r}{R}\right) - \frac{1}{4}\frac{d}{dz}\left(\frac{M - M_\alpha}{\xi}\right)\left(r^2 - R^2 - 2\xi^2\ln\frac{r}{R}\right) + \frac{dM_\alpha}{dz}\xi\ln\frac{r}{R}\right]. \qquad (7)$$

Here we have introduced the notation $M_\alpha = 2\pi\alpha/\mu_0 I$. We note that for $M_\alpha > M$ a cylindrical column of liquid becomes unstable [4], and therefore from now on we assume that $M_\alpha < M$.

Using (7) it follows from (6) that

$$\frac{\eta Q}{\mu_0 I} = \frac{dM}{dz}\left(\frac{7}{12}\xi^3 - \frac{1}{2}\xi^3\ln\frac{\xi}{R} - \frac{1}{2}R\xi^2 - \frac{1}{4}R^2\xi + \frac{1}{6}R^3\right) -$$
$$-\frac{1}{4}\frac{d}{dz}\left(\frac{M - M_\alpha}{\xi}\right)\left(\frac{3}{4}\xi^4 - \xi^4\ln\frac{\xi}{R} - R^2\xi^2 + \frac{1}{4}R^4\right) + \frac{1}{4}\frac{dM_\alpha}{dz}\xi\left(2\xi^2\ln\frac{\xi}{R} - \xi^2 + R^2\right). \qquad (8)$$

The solution of the first-order ordinary differential equation (8) gives the function $\xi(z)$, which determines the shape of the free surface. Functions $M(z)$ and $M_\alpha(z)$ are assumed given. By using (8) the derivative $d\xi/dz$ can be eliminated from Eq. (7) for the velocity profile in the film. The analysis and numerical calculations are more conveniently performed by introducing the dimensionless coordinates $r_1 = r/R$ and $\zeta = \xi/r$, where r_1 varies from 1 to ζ. Then

$$v = \frac{\mu_0 I\,(dM/dz)R}{2\pi\eta}\left\{r_1 - 1 + (\gamma - 1)\zeta\ln r_1 + \frac{r_1^2 - 1 - 2\zeta^2\ln r_1}{(3/4)\zeta^4 - \zeta^4\ln\zeta - \zeta^2 + 1/4}\times\right.$$
$$\left.\times\left[A - \frac{1}{2}(\gamma - 1)\zeta^3\ln\zeta + \frac{1}{4}\left(\gamma - \frac{7}{3}\right)\zeta^3 + \frac{1}{2}\zeta^2 - \frac{1}{4}(\gamma - 1)\zeta - \frac{1}{6}\right]\right\}, \gamma \equiv \frac{dM_\alpha/dz}{dM/dz}, \quad A \equiv \frac{\eta Q}{\mu_0 I\,(dM/dz)R^3}. \qquad (9)$$

The dimensionless velocity profile $V_1 = 2\pi\eta v/\mu_0 I(dM/dz)R$ calculated from this equation for dM/dz = const and dM_α/dz = const is shown in Fig. 1. In this case the thermocapillary (dM_α/dz) and thermomagnetic (dM/dz) mechanisms contribute equally to the excitation of the motion of the liquid ($\gamma = 1$). For $\gamma = 0$ the only effective mechanism is thermomagnetic, and for $\gamma = \infty$ thermocapillary. The velocity profiles V_1 and $V_2 = 2\pi\eta v/\mu_0 I(dM_\alpha/dz)R$ corresponding to these cases are shown in Fig. 2 for $\zeta = 2$ and zero flow rate of the liquid ($A = 0$). The velocity profile $V_3 = 2\pi R^2 v/Q$ corresponds to an isothermal liquid.

We first analyze the case of an isothermal liquid ($dM/dz = 0$, $dM_\alpha/dz = 0$) from the point of view of the solution of Eq. (8). Then the only mechanism moving the liquid is the drop of the surface level relative to the equilibrium level, and the equation of the free surface from (8) can be written

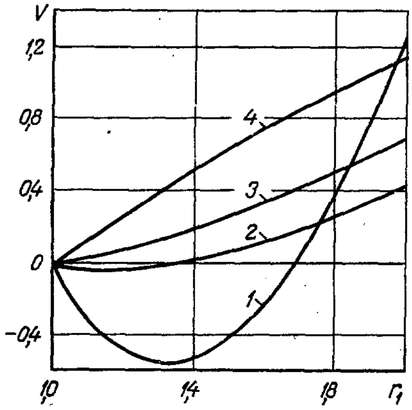

Fig. 1

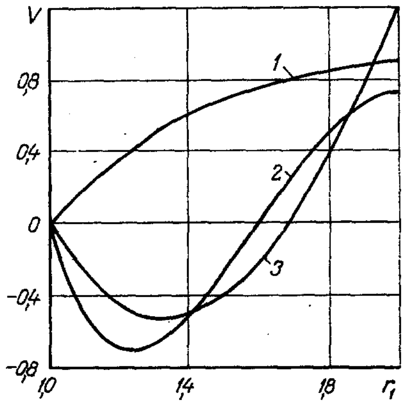

Fig. 2

Fig. 1. Profiles of dimensionless velocity V in film ($\zeta = 2$, $\gamma = 1$); 1) $A = 0$, $V = 5V_1$; 2) $A = 0.2$, $V = V_1$; 3) $A = 0.5$, $V = V_1$; 4) $A = 1$, $V = V_1$.

Fig. 2. Profiles of dimensionless velocity V in film ($\zeta = 2$): 1) $V = V_3$; 2) $\gamma = 0$, $A = 0$, $V = 10^2 V_1$; 3) $\gamma = \infty$, $A = 0$, $V = 5V_2$.

$$z = \frac{\mu_0 MIR^3}{4\eta Q}\left[\frac{13}{36}(\zeta^3 - \zeta_0^3) - \frac{1}{3}(\zeta^3 \ln\zeta - \zeta_0^3 \ln\zeta_0) - (\zeta - \zeta_0) - \frac{1}{4}\left(\frac{1}{\zeta} - \frac{1}{\zeta_0}\right)\right], \tag{10}$$

where ζ_0 is the value of the dimensionless coordinate of the free surface at the point $z = 0$.

For a thin film $\zeta - 1 \equiv \delta \ll 1$ this equation goes over naturally into the equation describing plane-parallel film flow of an ordinary liquid under the action of the force of gravity ρg perpendicular to the plane of flow:

$$z = \frac{\mu_0 MG(\xi_0 - R)^4}{12\eta Q_1}\left[1 - \left(\frac{\delta}{\delta_0}\right)^4\right], \tag{11}$$

where $Q_1 \equiv Q/2\pi R$ is the flow rate of the liquid per unit length of cross section, and in place of the gravitational force there appears the magnetic force $\mu_0 MG$, determined by the constant gradient of the magnetic field intensity G, equal in the present case to $I/2\pi R^2$. If $\zeta = \zeta_L$ is the value of the ordinate of the free surface of the film at the point $z = L$, the flow rate of the liquid Q is given in terms of the values of the levels ζ_0 and ζ_L by the expression

$$Q = \frac{\mu_0 MIR^3}{4\eta L}\left[\frac{13}{36}(\zeta_L^3 - \zeta_0^3) - \frac{1}{3}(\zeta_L^3 \ln\zeta_L - \zeta_0^3 \ln\zeta_0) - (\zeta_L - \zeta_0) - \frac{1}{4}\left(\frac{1}{\zeta_L} - \frac{1}{\zeta_0}\right)\right], \tag{12}$$

and for a thin film

$$Q = \frac{\mu_0 MG(\xi_0 - R)^4}{12\eta L}\left[1 - \left(\frac{\delta_L}{\delta_0}\right)^4\right]. \tag{13}$$

For a nonisothermal liquid we consider the closed ($Q = 0$) convective flow of a liquid resulting from the thermomagnetic and thermocapillary mechanisms for a thin film ($\delta \ll 1$).

In this case Eq. (8) becomes

$$\frac{M - M_\alpha}{3}\frac{d\delta^2}{dz} + \frac{1}{4}\frac{dM}{dz}\delta^2 = \frac{dM_\alpha}{dz} \tag{14}$$

which has the solution

$$\delta^2 = \frac{3}{\psi}\left[\int \frac{\psi}{M - M_\alpha}\, dM_\alpha + \mathrm{const}\right], \tag{15}$$

where $\psi \equiv \exp\left[\frac{3}{4}\int \frac{dM}{M - M_\alpha}\right]$.

A characteristic feature of the case considered is that even for a thin film constant curvature in a cross section affects the profile of the free surface [in (15) the difference $M - M_\alpha$]. Only when $M_\alpha << M$ do these equations go over into the relations describing plane-parallel flow [5]:

$$\mu_0 G (R\delta)^2 = 3M^{-3/4}[\int M^{-1/4} d\alpha + \text{const}].$$

Numerical estimates show that the magnetic force $\mu_0 MG$ may be an order of magnitude larger than the gravitational force ρg, and ensure a rather intense film flow of a magnetizable liquid.

NOTATION

H, magnetic field intensity; M, magnetization; μ_0, magnetic permeability of vacuum; I, current (r, φ, z), cylindrical coordinates; (ξ, ζ), coordinates of free surface; R, radius of current-carrying conductor; p, pressure; v, axial component of velocity; η, viscosity; R_1, R_2, principal radii of curvature of surface; α, surface tension; Q, flow rate of liquid; G, characteristic value of gradient of magnetic field intensity; ρ, density; g, acceleration due to gravity.

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